Seminário Brasileiro de Análise - SBA
Instituto de Matemática e Estatística - USP
Edição Nº 70 November 2009

INSTABILITY OF REDUCIBLE SOLUTIONS FOR THE SEIBERG-WITTEN EQUATIONS *

C. M. DORIA †

16/11/2009

Abstract

The reducible solutions for the SW-eq. may not be stable. They become unstable in the presence of a SW-monopole, however it is a open problem to show the existence of a SW-monopole. The purpose of this paper is to give the first step to estimate the instability of a reducible solution for the Seiberg-Witten equations by showing its Morse index is finite. The estimate is done by computing the 2^{nd}-variation formula for the Seiberg-Witten functional in a Morse Theory framework.

1 Introduction

Let (X, g) be a smooth, closed 4-manifold endowed with a riemannian metric g. For a fixed a spin^c structure on X, the Seiberg-Witten equations are

$$F_A^+ = \sigma(\phi) \tag{1.1}$$

$$D_A\phi = 0 \tag{1.2}$$

(i) D_A^+ is the $Spinc^c$-Dirac operator defined on $\Gamma(S_\alpha^+)$,
(ii) $\sigma : \Gamma(S_\alpha^+) \to End^0(S_\alpha^+)$ is the quadratic form $\sigma(\phi) = \phi \otimes \phi^* - \frac{|\phi|^2}{2}.I$

The Seiberg-Witten theory is a Gauge Theory. It is also a Topological Quantum Field Theory conjectured to be dual to the Yang-Mills Theory [12]. The expectation values of SW quantum theory are the Seiberg-Witten invariants of X. It is known from Quantum Field Theory that the classical solutions matters to the expectation values of the quantized theory. The reducible solutions are the classical solutions of type $(A, 0)$, so the eq. (1.2) is trivially satisfied and the eq. (1.1) becomes $F_A^+ = 0$. The existence of reducible solutions is a consequence of the existence of a harmonic 2-forms in each cohomology class of $H^2(X, \mathbb{R})$, so

* *Mathematics Subject Classifications:* 58E99, 81T45, 81T13
Key words: Seiberg-Witten, Gauge , Morse
†UFSC, Departamento de Matemática, SC, Brasil, cmdoria@mtm.ufsc.br